Paper ID: TE-02

Finite Difference Solution of the Transient Free Convection in Micropolar Fluid with Heat Generation and Constant Heat Flux

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Abstract

In this paper the numerical solution of the transient free convection in micropolar fluid with heat generation and constant heat flux is discussed. The corresponding governing equations are a set of coupled nonlinear partial differential equations. Here the system of the governing equations is solved numerically using the finite difference method. Before use of the finite difference method the governing equations are transferred to their non-dimensional form by introducing suitable substitutions. As a result, some physically interesting non-dimensional numbers, viz. Prandtl number and Grasshof number, as well as some non-dimensional material parameters are aroused. The non-dimensional equations are then discretized and solved. The stability criterion of the solutions is considered, though is included due to space constraint. The numerical solutions thus obtained at different grid points at different time is plotted and discussed. The values proportional to the coefficient of skin friction and Nusselt number are also tabulated and discussed.

Keywords: *Finite difference method, Transient flow, Free Convection, Micropolar Fluid.*

1. Introduction

The concept of micropolar fluids introduced by Eringen [1] deals with a class of fluids which exhibit certain microscopic effects arising from the local structure and micromotions of the fluid elements. These fluids contain dilute suspensions of rigid macromolecules with individual motions which support stress and body moments and are influenced by spin-inertia. Physically micropolar fluids represent fluids consisting of randomly oriented particles suspended in a viscous medium, where the deformation of fluid particles is ignored It has found its applications specially, in lubrication theory. Zakaria [2] pointed that the theory of micropolar fluid and its extension to thermo-micropolar fluids may form suitable non-Newtonian fluid models which can be used to analyze the behavior of exotic lubricants, colloidal suspensions or polymeric fluids, liquid crystals and animal blood. Some theoretical studies have been compared and favorably agree with experimental measurement.

Natural convection with internal heat generation finds application in fire and combustion modeling. Gorla and Tornabene [3] investigated the effects of thermal radiation on mixed convection flow over a vertical plate with non-uniform heat flux boundary condition. Raptis [4] studied numerically the case of a steady two dimensional flow of a micropolar fluid past a continuously moving plate with a constant velocity in the presence of thermal radiation. Kim [5] studied the unsteady free convection flow of a micropolar fluid through a porous medium bounded by an infinite vertical plate. Kim and Fedorov [6] studied the transient mixed radiative convection flow of a micropolar fluid past a moving, semi-infinite vertical porous plate. El-Amin [7] studied the combined effect of internal heat generation and magnetic field on free convection and mass transfer flow in a micropolar fluid with constant suction. El-Hakiem [8] studied the natural convection in a micropolar fluid with thermal dispersion and internal heat generation.

2. Governing Equations and their manipulation

Let us consider the free convection of a micropolar fluid along the vertical plate. The temperature of the plate is held at constant value of T_s and the heat flux is considered as constant, the thermal dispersion effect is also included. Let the *x*-axis is along the plate in the vertical direction and *y*-axis is perpendicular to the plate. The governing equations with the Boussinesq approximation can be put in the following form

1) Mass equation: $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$

2) Momentum equation: $\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=(\nu+\frac{k}{\rho})\frac{\partial^2 u}{\partial y^2}+g^*\beta(T-T_\infty)+\frac{k}{\rho}\frac{\partial N}{\partial y}$

3) Angular momentum equation: $\frac{\partial N}{\partial t}+u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=\frac{\gamma}{\rho j}\frac{\partial^2 N}{\partial y^2}-\frac{k}{\rho j}(2N+\frac{\partial u}{\partial y})$

4) Energy equation: $\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k'}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+Q(T-T_\infty)$

With the boundary conditions: $u(x,0,t)=0,\ N(x,0,t)=0,\ T(x,0,t)=T_s$

$$u(x,\infty,t)=0,\ N(x,\infty,t)=0,\ T(x,\infty,t)=T_\alpha,$$

Here u and v are velocity components associated with x and y directions measured along and normal to the vertical plate respectively, ν the kinematic coefficient of viscosity, k the vortex viscosity, ρ the density of the fluid, g^* the acceleration due to gravity, β the coefficient of thermal expansion, T the temperature of the fluid in the boundary layer, T_α the free steam temperature, N the angular velocity, γ the spin gradient viscosity, j the microinertia per unit mass, k' the thermal conductivity, c_p specific heat at constant pressure and Q the heat generation.

The variables are made dimensionless with the following substitutions.

$$u^*=\frac{u}{U_0},\ v^*=\frac{v}{U_0},\ x^*=\frac{U_0}{\nu}x,\ y^*=\frac{U_0}{\nu}y,\ t^*=\frac{U_0^2}{\nu}t,\ N^*=\frac{\nu}{U_0^2}N,\ j^*=\frac{U_0^2}{\nu^2}j,\ \theta=\frac{(T-T_\infty)}{(T_s-T_\infty)}$$

Along with them the following dimensionless quantities are introduced.

Prandtl Number, $P_r=\frac{\upsilon}{k/\rho C_p}=\frac{\upsilon\rho c_p}{k}$, Grashof number, $Gr=\frac{\nu g^*\beta(T_s-T_\infty)}{U_0^3}$

Dimensionless material parameters $\lambda=\frac{\gamma}{\rho\nu j}$, and $\Delta=\frac{k}{\rho\nu}$

On substitution of the dimensionless variables the Momentum equation becomes

$$\frac{\partial u^*}{\partial t^*}+u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=(1+\Delta)\frac{\partial^2 u^*}{\partial y^{*2}}+G_r\theta+\Delta\frac{\partial N^*}{\partial y^*}$$

After dropping the asterisks, we have

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=(1+\Delta)\frac{\partial^2 u}{\partial y^2}+\Delta\frac{\partial N}{\partial y}+G_r\theta \quad (1)$$

Similarly the Angular momentum equation takes the following form

$$\frac{\partial N^*}{\partial t^*}+u^*\frac{\partial N^*}{\partial x^*}+v^*\frac{\partial N^*}{\partial y^*}=\lambda\frac{\partial^2 N^*}{\partial y^{*2}}-\frac{\Delta}{j^*}(2N^*+\frac{\partial u^*}{\partial y^*})$$

After dropping the asterisks, we have

$$\frac{\partial N}{\partial t}+u\frac{\partial N}{\partial x}+v\frac{\delta N}{\partial y}=\lambda\frac{\partial^2 N}{\partial y^2}-\frac{\Delta}{j}(2N+\frac{\partial u}{\partial y}) \quad (2)$$

And the Energy equation become

$$\frac{\partial\theta}{\partial t^*}+u^*\frac{\partial\theta}{\partial x^*}+v^*\frac{\partial\theta}{\partial y^*}=\frac{1}{P_r}\frac{\partial^2\theta}{\partial y^{*2}}+\alpha\theta$$

where, $\alpha=\frac{\nu}{U_0{}^2}Q$

After dropping the asterisks, we have

$$\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{1}{P_r}\frac{\partial^2\theta}{\partial y^2}+\alpha\theta \tag{3}$$

The transferred boundary conditions are:

$$u(x,0,t)=0,\ \ N(x,0,t)=0,\ \ \theta(x,0,t)=1$$
$$u(x,\infty,t)=0,\ \ N(x,\infty,t)=0,\ \ \theta(x,\infty,t)=0,$$

It is shown in [9] that the skin-friction coefficient c_f is proportional to $\frac{1}{U_0^2}[1+\Delta]\nu\left(\frac{\partial u}{\partial y}\right)_{y=0}$

For simplicity an explicit method is used to solve the obtained non-dimensional governing equations. Let u', N', θ' denote the values of u, N and θ at the end of a time step. Then the appropriate finite difference equations corresponding to equations (1), (2), and (3) are

$$\frac{u'_{ij}-u_{ij}}{\delta\tau}+u_{ij}\frac{u_{ij}-u_{i-1j}}{\delta X}+v_{ij}\frac{u_{ij+1}-u_{ij}}{\delta Y}=(1+\Delta)\frac{u_{ij+1}-2u_{ij}+u_{ij-1}}{(\delta Y)^2}+\Delta\frac{N_{ij+1}-N_{ij}}{\delta Y}+G_r\theta \tag{4}$$

$$\frac{N'_{ij}-N_{ij}}{\delta\tau}+u_{ij}\frac{N_{ij}-N_{i-1j}}{\delta X}+v_{ij}\frac{N_{ij+1}-N_{ij}}{\delta Y}=\lambda\frac{N_{ij+1}-2N_{ij}+N_{ij-1}}{(\delta Y)^2}-\frac{\Delta}{j}(2N_{ij}+\frac{u_{ij+1}-u_{ij}}{\delta Y}) \tag{5}$$

$$\frac{\theta'_{ij}-\theta_{ij}}{\delta\tau}+u_{ij}\frac{\theta_{ij}-\theta_{i-1j}}{\delta X}+v_{ij}\frac{\theta_{ij+1}-\theta_{ij}}{\delta Y}=\frac{1}{P_r}\frac{\theta_{ij+1}-2\theta_{ij}+\theta_{ij-1}}{(\delta Y)^2}+\alpha\theta_{ij} \tag{6}$$

during any time step. The coefficients u_{ij}, v_{ij} appearing in (4), (5) and (6) are generally considered as constant. Then at the end of any time step $\delta\tau$, the new temperature θ' the new angular momentum N' and the new velocity components u' at all interior grid points may be obtained by successive applications of (6), (5) and (4) respectively. This process is repeated in time provided the time step is sufficiently small and u, N, θ should eventually converge to values which approximate the steady state solution of equations (1), (2) and (3).

3. Results and Discussion

In this paper, the effect of transient free convection on micropolar fluid with heat generation and constant heat flux has been investigated using the finite difference method. To study the physical situation of this problem, the numerical values of the velocity, temperature and angular momentum within the boundary layer has been computed and also the coefficients proportional to the skin friction coefficient and Nusselt number are calculated. It is seen that the solution has dependency on heat source parameter α, micro inertia per unit mass j, dimensionless material parameter Δ, the Grashof number Gr, dimensionless material parameter λ and the Prandtl number Pr. Due to space constraint effect of all the parameters is not presented here. The values 0.2, 0.5, 0.71, 0.73, 1, 2, 5, 7.01 are considered for Pr and the values of other parameters are however chosen arbitrarily.

Figures 1(a)-1(c) show the velocity, temperature and angular momentum profiles for different values of heat source parameter α respectively. From Fig. 1(a) and 1(b) it is observed that the effect of α on velocity and temperature is similar. They increase with the increase in α rapidly but within short distance become zero. In Fig. 1(c), with the increase in α, both the negative and positive values of angular momentum increases. When α is large the negative zone of the angular momentum is small in comparison to the small α. For large value of α the angular momentum oscillates more from negative to positive.

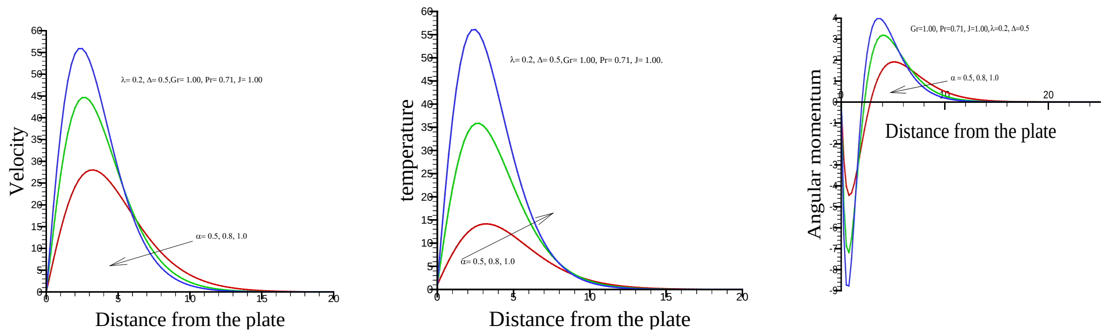

Fig. 1. Velocity, Temperature and Angular Momentum profiles for different values of heat source parameter, α

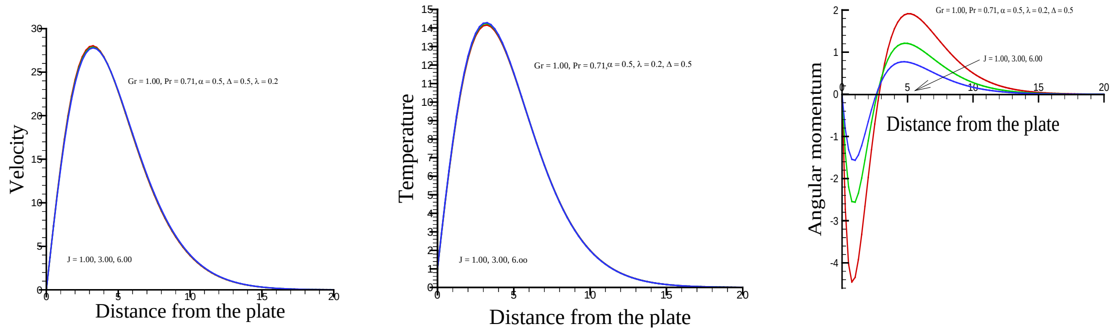

Fig. 2. Velocity, Temperature and Angular Momentum profiles for different values of micro inertia per unit mass, j

Figures 2(a)-2(c) are showing the velocity, temperature and angular momentum profiles for different values of micro inertia per unit mass, j respectively. From figure 2(a) and 2(b), there is a very small change in velocity and temperature due to increase the value of j. They spread a little with the increase of j and within a short distance becoming zero. The magnitude of the angular momentum, both negative and positive, decreases with the increase in j. Clearly j has dominant effect over angular momentum, but has less impact on velocity and temperature.

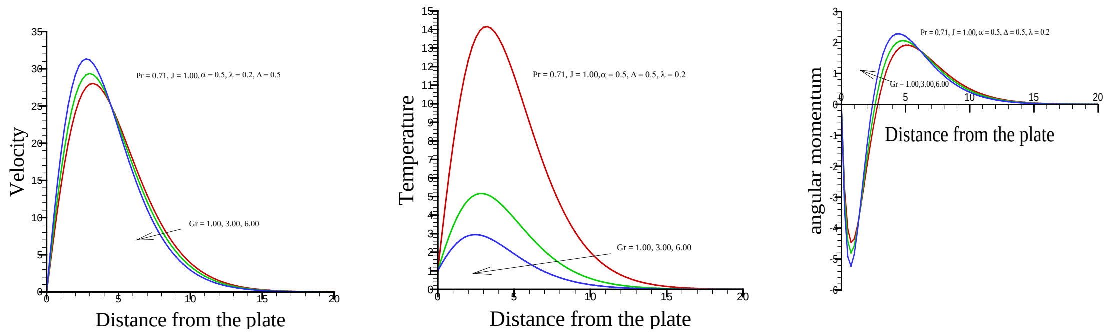

Fig. 3. Velocity, Temperature and Angular Momentum profiles for different values of the Grashof number, Gr

For different values of Grashof Number Gr, the profiles for velocity, temperature and angular momentum are presented in Fig 3(a)-3(c). In Fig. 3(a), we observed that with the increase in Gr value of the maximum velocity increases whereas the spreading of velocity decreases. From Fig. 3(b), it is found that the temperature decreases with the increase in Gr. Also the temperature is becoming zero far away from the plate for lower values of Gr. The angular momentum fluctuation from negative to positive increases with the increase in Gr and for lower values of Gr though the magnitude of angular momentum are less but its spreading is more.

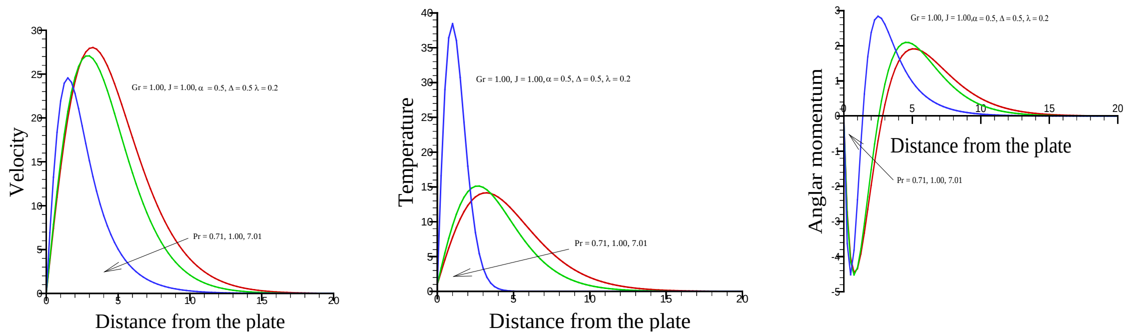

Fig. 4. Velocity, Temperature and Angular Momentum profiles for different values of the Prandtl number, Pr

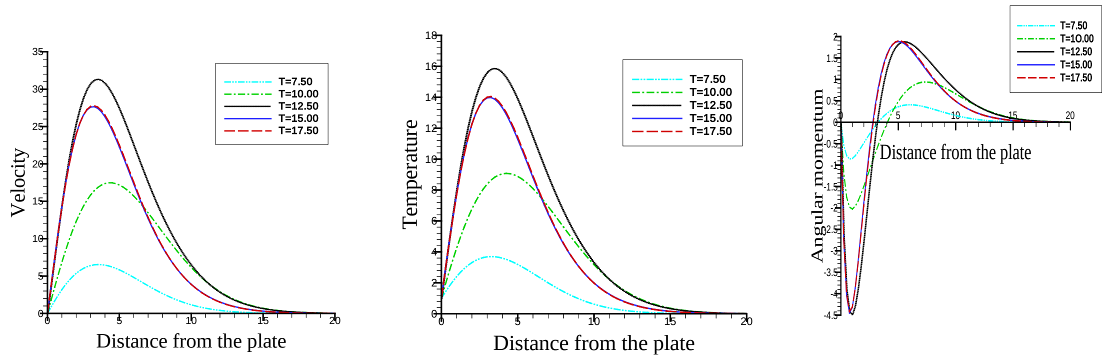

Fig. 5. Velocity, Temperature and Angular momentum profiles for different values of time, T

The profiles for velocity, temperature and angular momentum for different values of Prandtl number Pr are shown respectively in Figures 4(a)-4(c). The velocity is not only decreasing with the increase in Pr but also it is becoming zero within a short distance from the wall. The temperature profile squeezes with the increase in Pr but the pick value of the temperature rises sharply. The negative value of angular momentum near the wall has no appreciable change with the increase in Pr. But positive values of angular momentum increases with Pr and its effective zone decreases.

In Table 1, the values proportional to the coefficient of skin friction and Nusselt numbers for different values of α are tabulated with fixed values of the other parameters. From the table it is observed that with the increase in α the value proportional to the skin friction increases, whereas in the case of Nusselt number the situation is reversed. Also the rate of decrease in Nusselt number is far more than the rate of increase.

In Table 2, the same is tabulated as table (4.1), but different values of microinertia j with the other parameters are kept unchanged. It is seen that the microinertia has a very little impact on the coefficient of skin friction and Nusselt numbers.

In Table 3, the values proportional to the skin-friction coefficient and the Nusselt number are tabulated against the Grashhof number Gr. It is seen that both of them increases with the increase in Gr. For skin friction the rate of increase is almost same, but in the case of Nusselt number the rate slows down for higher values of Gr.

Finally the effect of Prandtl number, Pr on the coefficient of skin friction and Nusselt numbers are presented in Table 4. Here again, as usual, the other parameters are kept as constant. It is seen that with the increase in Pr the value proportional to skin friction increases, whereas the Nusselt number decreases.

Table 1, Numerical values proportional to skin friction coefficient C_f and Nusselt number Nu for different values of α, taking $Gr = 1.00, \Pr = 0.71, j = 1.00, \lambda = 0.2, \Delta = 0.5$ as fixed

α	Values proportional to C_f	Values proportional to Nu
0.5	9.822492	-4.726112
0.8	18.620130	-14.733240
1.0	25.665870	-25.549920
1.5	46.543100	-69.980420
2.0	71.292650	-143.382300
2.5	99.310490	-250.198700

Table 2, Numerical values proportional to skin friction coefficient C_f and Nusselt number Nu for different values of j, taking $Gr = 1.00, \Pr = 0.71, \alpha = 0.5, \lambda = 0.2, \Delta = 0.5$ as fixed.

j	Values proportional to C_f	Values proportional to Nu
1.00	9.822492	-4.726112
2.00	9.792691	-4.752003
3.00	9.788363	-4.766579
4.00	9.788019	-4.775903
5.00	9.788466	-4.782364
6.00	9.789032	-4.787105

Table 3, Numerical values proportional to skin friction coefficient C_f and Nusselt number Nu for different values of Gr , taking $\Pr = 0.71, j = 1.00, \alpha = 0.5, \lambda = 0.2, \Delta = 0.5$ as fixed.

Gr	Values proportional to C_f	Values proportional to Nu
1.00	9.822492	-4.726112
2.00	11.018580	-2.537850
3.00	12.156370	-1.796058
4.00	13.238690	-1.416348
5.00	14.273790	-1.182411
6.00	15.268690	-1.022038

Table 4, Numerical values proportional to skin friction coefficient C_f and Nusselt number Nu for different values of Pr, taking $Gr = 1.00, j = 1.00, \alpha = 0.5, \lambda = 0.2 \Delta = 0.5$ as fixed.

Pr	Values proportional to C_f	Values proportional to Nu
0.20	7.927539	-2.109994
0.71	9.822492	-4.726112
0.73	9.869625	-4.819553
1.00	10.446450	-6.076511
5.00	16.005450	-27.739200
7.01	18.124500	-40.557210

4. Conclusion

In this paper finite difference technique has been used to solve the governing partial coupled nonlinear differential equations of the transient free convection flow of a micropolar fluid. It is found that if the stability criterion of the time integration is satisfied then the obtained result reflects the behaviour of the flow with respect to the different flow parameters. In these type system when there are heat generation and constant heat flux then

i) With the increase in the heat source parameter velocity, temperature and angular momentum increases.

ii) With the increase in the Prandtl number the effective zone of change of velocity, temperature and angular momentum reduces.

iii) Temperature profile shows greater sensitivity of Grashof number and the Nusselt number increases with the increase in the Grashof number.

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