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Heat absorption and Joule heating effect along a vertical wavy surface on MHD free convection flow with viscosity dependent on temperature

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Abstract

The interaction of free convection with heat absorption and Joule heating of viscous incompressible fluid and heat transfer characteristics on MHD steady two-dimensional laminar flow along a uniformly heated vertical wavy surface with viscosity dependent on temperature has been analyzed numerically. Using the appropriate variables; the basic equations are transformed to convenient form. The resulting nonlinear systems of partial differential equations are mapped into the domain of a vertical flat plate. The governing equations are solved numerically using an implicit finite difference scheme of Keller-box type. The effect of various physical parameters such as the heat absorption parameter, Joule heating parameter and viscosity parameter on the streamline and isotherms of the fluid as well as the shearing stress and heat transfer rate in terms of the skin friction coefficient and local Nusselt number are computed and presented graphically in detail while, magnetic parameter (M) and the amplitude-to-length ratio of the wavy surface (α) are considered fixed. It is found that the numerical results are strongly dependent on the set of parameters entering into the problem.

Keywords: *Heat absorption, Joule heating, temperature dependent viscosity, MHD, free convection, Keller-box method, wavy surface.*

1. Introduction

The study of heat generation or absorption in moving fluids is important in problems dealing with chemical reactions and those concerned with dissociating fluids. The characteristics of free convection flow of electrically conducting fluid in the presence of heat absorption on magnetic field along a wavy surface is important from the technical point of view and such type of problems have received much attention of many researchers. If the surface is roughened the flow is disturbed by the surface and this alters the rate of heat transfer. For examples, flat-plate solar collectors, flat-plate condensers in refrigerators and heat exchanger. The viscosity of the fluid to be proportional to a linear function of temperature, two semi-empirical formulae which was proposed by Charraudeau [1]. Yao [2] first investigated the free convection heat transfer from an isothermal vertical wavy surface and used an extended Prantdl's transposition theorem and a finite-difference scheme. He proposed a simple transformation to study the free convection heat transfer for an isothermal vertical sinusoidal surface. These simple coordinate transformations method to change the wavy surface into a flat plate. Vajravelu and Hadjinolaou [3], studied the heat transfer characteristics in the laminar boundary layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation. In this study they considered that the volumetric rate of heat generation, q^m [W /m^3], should be $q^m = Q_0(T- T_\infty)$, for $T \geq T_\infty$ and equal to zero for $T < T_\infty$, where Q_0 is the heat generation/absorption constant. The above relation explained by Vajravelu and Hadjinolaou [3], is valid as an approximation of the state of some exothermic process and having T_∞ as the onset temperature. When the inlet temperature are not less than T_∞ they used $q^m = Q_0(T- T_\infty)$. Alam et al. [4] considered the problem of free convection from a wavy vertical surface in presence of a transverse magnetic field using Keller box method. The combined effects of thermal and mass diffusion on the natural convection flow of a viscous incompressible fluid along a vertical wavy surface investigated by Hossain and Rees [5]. Cheng [6] studied the natural convection heat and mass transfer near a vertical wavy surface with constant wall temperature and concentration in a porous medium. The problem of natural convection of fluid with temperature dependent viscosity along a heated vertical wavy surface have been studied by Hossain et al. [7]. Molla et al. [8] numerically investigated natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation/absorption. Molla et al. [9] also studied radiation effect on mixed convection laminar flow along a vertical wavy surface. Very recently, Parveen and Alim [10] considered the effect of MHD free convection flow in presence of Joule heating and heat generation with viscosity dependent on temperature along a vertical wavy surface.

In all the aforementioned analysis the heat absorption and Joule heating effect on magnetic field with temperature dependent viscosity free convection flow along wavy surface have not been studied. The current study is used to deal with this problem. Using the appropriate transformations, the boundary layer equations are reduced to dimensionless partial differential forms. Numerically results have been obtained in terms of local skin friction and the rate of heat transfer in terms of local Nusselt number, the streamlines as well as the isotherms for a selection of relevant physical parameters and discussed graphically.

2. Mathematical formulation of the problem

The boundary layer analysis outlined below allows $\overline{\sigma}(X)$ being arbitrary, but our detailed numerical work assumed that the surface exhibits sinusoidal deformations. The wavy surface may be described by

$$Y_w = \overline{\sigma}(X) = \alpha \sin\left(\frac{n\pi X}{L}\right) \tag{1}$$

where L is the wave length associated with the wavy surface.

The geometry of the wavy surface and the two-dimensional cartesian coordinate system are shown in Fig. 1.

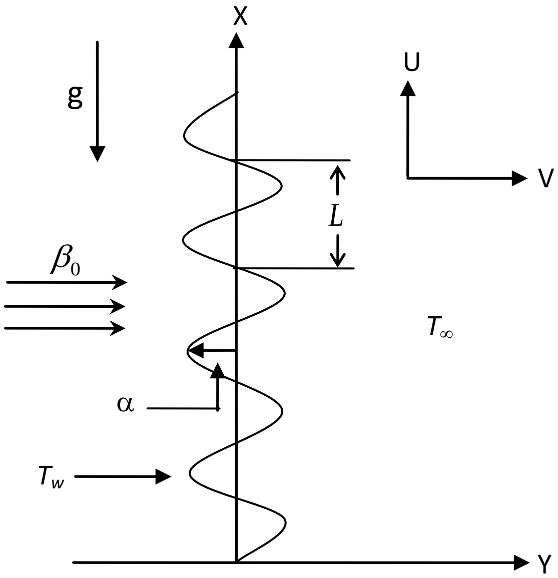

Fig. 1. Physical model and coordinate system

Under the usual Boussinesq approximation, the governing equations describing the conservation of mass, momentum and energy, respectively can be written non dimensional form as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + Gr^{1/4}\sigma_x \frac{\partial p}{\partial y} + \left(1+\sigma_x^2\right)(1+\varepsilon\theta)\frac{\partial^2 u}{\partial y^2} + \varepsilon\left(1+\sigma_x^2\right)\frac{\partial \theta}{\partial y}\frac{\partial u}{\partial y} - Mu + \theta \tag{3}$$

$$\sigma_x\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -Gr^{1/4}\frac{\partial p}{\partial y} + \sigma_x\left(1+\sigma_x^2\right)(1+\varepsilon\theta)\frac{\partial^2 u}{\partial y^2} + \varepsilon\sigma_x\left(1+\sigma_x^2\right)\frac{\partial \theta}{\partial y}\frac{\partial u}{\partial y} - \sigma_{xx}u^2 \tag{4}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\left(1+\sigma_x^2\right)\frac{\partial^2 \theta}{\partial y^2} + Q\theta + Ju^2 \tag{5}$$

In the above equations Pr, Q, ε, J and M are respectively known as the Prandtl number, heat absorption parameter, viscosity variation parameter, Joule heating parameter and magnetic parameter, which are defined as

$$\Pr = \frac{C_p \mu_\infty}{k},\ Q = \frac{Q_0 L^2}{\mu C p Gr^{1/2}},\ \varepsilon = \varepsilon^*(T_w - T_\infty),\ J = \frac{\sigma_0\ \beta_0^2\ \nu\, Gr^{1/2}}{\rho C_p\left(T_w - T_\infty\right)} \text{ and } M = \frac{\sigma_0 \beta_0^2 L^2}{\mu Gr^{1/2}}$$

The variable viscosity chosen in this study which is introduced by Charraudeau [1] and used by Hossain et al. [7] as follows:

$$\mu = \mu_\infty[1+\varepsilon^*(T-T_\infty)] \tag{6}$$

where μ_∞ is the viscosity of the ambient fluid and ε^* is a constant.
Following Yao [2], here introduce the following non-dimensional variables

$$x = \frac{X}{L},\ y = \frac{Y-\overline{\sigma}}{L} Gr^{\frac{1}{4}},\ u = \frac{\rho L}{\mu_\infty} Gr^{-1/2} U,\quad v = \frac{\rho L}{\mu_\infty} Gr^{-1/4}\left(V - \sigma_x U\right)$$

$$\theta = \frac{T-T_\infty}{T_w - T_\infty}, \sigma_x = \frac{d\overline{\sigma}}{dX} = \frac{d\sigma}{dx}, Gr = \frac{g\beta(T_w - T_\infty)}{\nu^2} L^3,\ p = \frac{L^2}{\rho \nu^2} Gr^{-1} P$$

Equation (4) indicates that the pressure gradient along the *y*-direction is $O(Gr^{-1/4})$, which implies that lowest order pressure gradient along *x*-direction can be determined from the inviscid flow solution. For the present problem this pressure gradient ($\partial p/\partial x = 0$) is zero. Equation (4) further shows that $Gr^{1/4}\partial p/\partial y$ is O(1) and is determined by the left-hand side of this equation. Thus, the elimination of $\partial p/\partial y$ from equations (3) and (4) leads to

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \left(1+\sigma_x^2\right)(1+\varepsilon\theta)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma_x \sigma_{xx}}{1+\sigma_x^2}u^2 + \varepsilon\left(1+\sigma_x^2\right)\frac{\partial u}{\partial y}\frac{\partial \theta}{\partial y} - \frac{M}{1+\sigma_x^2}u + \frac{1}{1+\sigma_x^2}\theta \tag{7}$$

The corresponding boundary conditions for the present problem are

$$\left.\begin{array}{ll} u = v = 0,\quad \theta = 1 & at\quad y = 0 \\ u = \theta = 0,\quad p = 0 & as\quad y \to \infty \end{array}\right\} \tag{8}$$

Now we introduce the following transformations to reduce the governing equations to a convenient form:

$$\psi = x^{3/4} f(x,\eta),\quad \eta = yx^{-1/4},\quad \theta = \theta(x,\eta) \tag{9}$$

where η is the pseudo similarity variable and ψ is the stream function.
Introducing the transformations given in equation (9) into equations (7) and (5) the momentum and energy equations are transformed the following forms,

$$\left(1+\sigma_x^2\right)(1+\varepsilon\theta) f''' + \frac{3}{4} ff'' - \left(\frac{1}{2} + \frac{x\sigma_x\sigma_{xx}}{1+\sigma_x^2}\right) f'^2 + \frac{1}{1+\sigma_x^2}\theta - \frac{Mx^{1/2}}{1+\sigma_x^2} f' + \varepsilon\left(1+\sigma_x^2\right)\theta' f'' = x\left(f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right) \tag{10}$$

$$\frac{1}{\Pr}\left(1+\sigma_x^2\right)\theta'' + \frac{3}{4} f\theta' + x^{1/2} Q\theta + J\quad x^{3/2} f'^2 = x\left(f'\frac{\partial \theta}{\partial x} - \theta'\frac{\partial f}{\partial x}\right) \tag{11}$$

The boundary condition (8) now takes the following form:

$$\left.\begin{array}{ll} f(x,o) = f'(x,o) = 0, & \theta(x,o) = 1 \\ f'(x,\infty) = 0, & \theta(x,\infty) = 0 \end{array}\right\} \tag{12}$$

The rate of heat transfer in terms of the local Nusselt number, Nu_x and the local skin friction coefficient, C_{fx} take the following forms:

$$Nu_x (Gr/x)^{-1/4} = -\sqrt{1+\sigma_x^2}\,\theta'(x,o) \tag{13}$$

$$C_{fx}(Gr/x)^{1/4}/2 = (1+\varepsilon)\sqrt{1+\sigma_x^2}\ f''(x,o) \tag{14}$$

3. Method of solution

The governing equations are solved numerically with the help of implicit finite difference method together with the Keller-Box scheme [11]. The discretization of momentum and energy equations are carried out with respect to non-dimensional coordinates *x* and η to convey the equations in finite difference form by approximating the functions and their derivatives in terms of central differences in both the coordinate directions. Then the required equations are to be linearized by using the Newton's Quasi-linearization method. The linear algebraic equations can be written in a block matrix which forms a coefficient matrix. The whole procedure namely reduction to first order followed by central difference approximations, Newton's Quasi-linearization method and the block Thomas algorithm, is well known as Keller-box method.

4. Results and discussion

Here we have investigated numerically heat absorption and Joule heating effect of viscous incompressible fluid on MHD two-dimensional laminar flow along a uniformly heated vertical wavy surface with temperature dependent viscosity. In simulation, the values of heat absorption parameter are considered to be 0.0 to -4.0, Joule heating parameter ranging from 0.0 to 0.2 and viscosity parameter ranging from 0.0 (constant viscosity) to 15.0 while magnetic parameter $M = 0.1$, the amplitude-to-length ratio of the wavy surface $\alpha = 0.3$ and Prandtl number $\Pr = 0.7$ which corresponds to the air at 2100^0K and $\Pr = 0.0288$ which corresponds to the mercury at 0^0C respectively. The numerical results have been obtained for the set of parameters entering into the problem and presented graphically.

The local skin friction coefficient C_{fx} and the rate of heat transfer in terms of the local Nusselt number Nu_x for the effect of heat absorption parameter Q against x are plotted in Fig. 2 while $\alpha = 0.3$, $M = 0.1$, $\varepsilon = 5.0$, $J = 0.05$ and $\Pr = 0.0288$. It is noted that the skin friction coefficient significant decreases along the downstream direction of the surface and the rate of heat transfer from the surface increases with the increase of the heat absorption parameter Q. Heat absorption mechanism creates a layer of cold fluid adjacent to the heated surface and finally the temperature of the fluid decreases. For this reason the temperature gradient that is heat transfer rate in terms of the local Nusselt number from the surface increases. Owing to the lessening temperature, the viscosity of the fluid decreases and the corresponding local skin friction coefficient decreases.

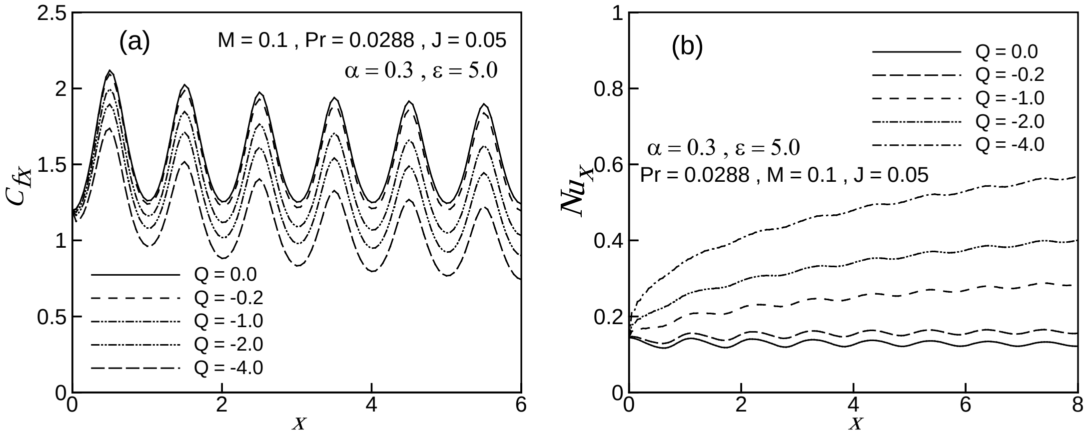

Fig. 2. Effect of Q on (a) skin friction coefficient C_{fx} and (b) rate of heat transfer Nu_x.

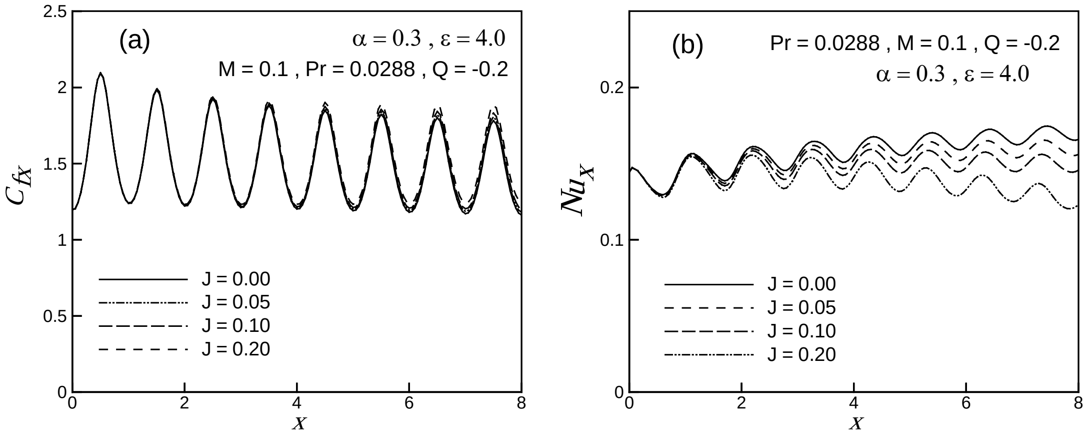

Fig. 3. Effect of J on (a) skin friction coefficient C_{fx} and (b) rate of heat transfer Nu_x.

The effect of variation of Joule heating parameter on the skin friction coefficient C_{fx} and local rate of heat transfer Nu_x are displayed in Fig. 3. The skin friction coefficient against x increases slowly along the upstream direction of the surface. On the other hand, the opposite situation observed for the rate of heat transfer. The highest values of local skin friction coefficient are recorded to be 2.09234 and 2.09576 for $J = 0.0$ and 0.2 respectively. The maximum values of local rate of heat transfer are 0.17824 and 0.15550 for $J = 0.0$ and 0.2 respectively. Finally, it is seen that the local skin friction coefficient increases by approximately 0.16% and the rate of heat transfer decreases by approximately 13% which occurs at the different position of x.

Fig. 4(a) and fig. 4(b) deal with the effect of viscosity parameter ε within the boundary layer for different values of the controlling parameters $Q = -0.01$, $M = 0.1$, $\alpha = 0.3$, $J = 0.02$ and $\Pr = 0.7$. It is observed that an increase in the value of variable viscosity parameter ε, the skin friction coefficient increases along the upstream direction of the surface and to decrease of the heat transfer rates. It is observed that the local skin friction coefficient increases by approximately 57% and the rate of heat transfer devalues by approximately 33% as ε changes from 0.0 to 15.0.

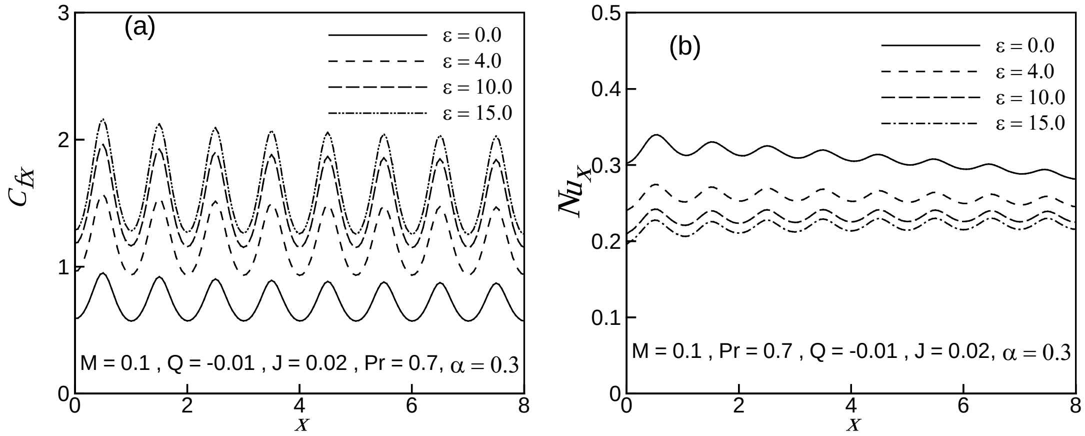

Fig. 4. Effect of ε on (a) skin friction coefficient C_{fx} and (b) rate of heat transfer Nu_x.

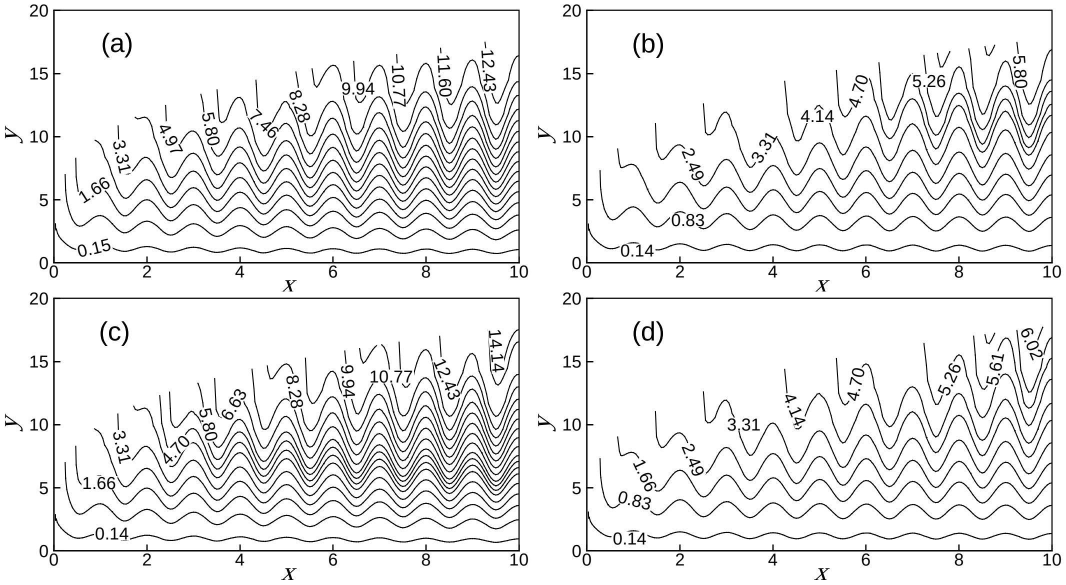

Fig. 5. Streamlines for (a) $Q = 0.0$, $J = 0.0$ (b) $Q = -4.0$, $J = 0.0$ (c) $Q = 0.0$, $J = 0.2$ (d) $Q = -4.0$, $J = 0.1$ while $\Pr = 0.0288$, $M = 0.1$, $\varepsilon = 5.0$ and $\alpha = 0.3$.

The effects of heat absorption parameter Q and Joule heating parameter J, on the development of streamlines which are illustrated in Fig. 5 for the amplitude-to-length ratio of the wavy surface $\alpha = 0.3$, $M = 0.1$, $\varepsilon = 5.0$ and Prandtl number $\Pr = 0.0288$. When $Q = 0.0$ and $J = 0.0$, where Joule heating effect is neglected and in absence of internal heat absorption as shown in Fig. 5(a) and found that maximum value of stream function ψ_{max} is 12.43. Fig. 5(b) indicates that the effect of heat absorption parameter strongly affect the velocity of the fluid flow and leads to thinner the velocity boundary layer. In this case the maximum value of stream function ψ_{max} is 5.80. From Fig. 5(c), it is observed that when the effect of Joule heating ($J = 0.2$) is considered the boundary layer becomes thicker. This is because Joule heating is the heating effect of conductors carrying currents. So velocity of the fluid flow increases and the maximum value of stream function ψ_{max} is 14.14 that is shown in Fig. 5(c). The combined effect of Q and J, are shown in Fig. 5(d). Here the maximum value of ψ_{max} is 6.02. From these figures it is observed that the value of stream function ψ becomes higher for the effect of Joule heating parameter J and ψ becomes smaller in presence of heat absorption parameter Q as well.

The variation of isotherms with heat absorption parameter Q and Joule heating parameter J for $\alpha = 0.3$, $M = 0.1$, $\varepsilon = 5.0$ and $\Pr = 0.0288$ are shown in Fig. 6. We can say after observing the isotherms of this figure that temperature enhances within the boundary layer due to the higher values of *J*. On the other hand, an opposite situation is observed on the temperature field within the boundary layer in the case of heat absorption.

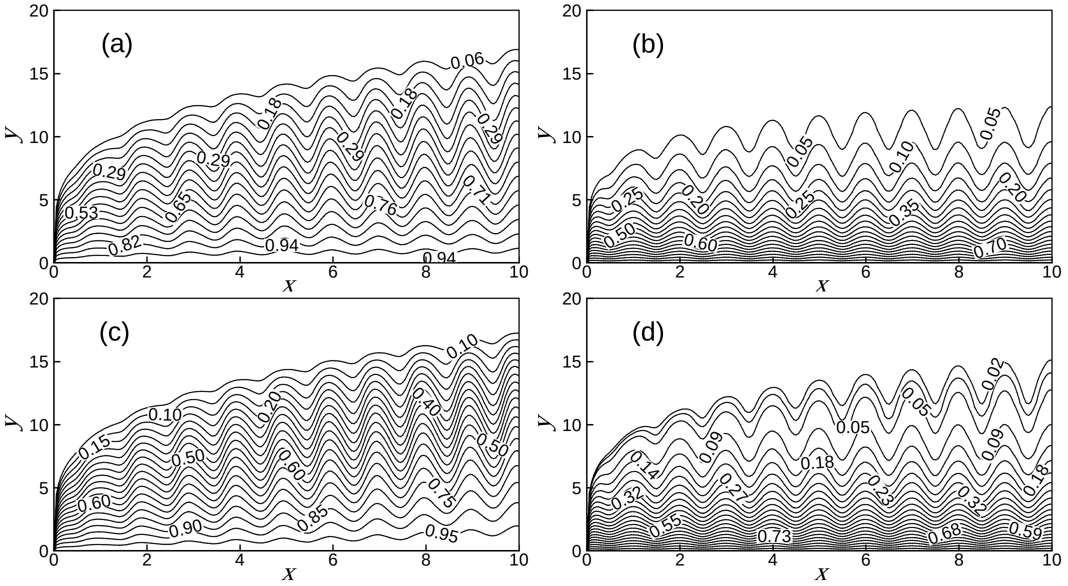

Fig. 6. Isotherms for (a) $Q = 0.0$, $J = 0.0$ (b) $Q = -4.0$, $J = 0.0$ (c) $Q = 0.0$, $J = 0.2$ (d) $Q = -4.0$, $J = 0.1$ while $\Pr = 0.0288$, $M = 0.1$, $\varepsilon = 5.0$ and $\alpha = 0.3$.

5. Conclusion

The problem of free convection heat transfer of viscous incompressible fluid with heat absorption and Joule heating on MHD two-dimensional laminar flow along a uniformly heated vertical wavy surface including viscosity dependent on temperature has been analyzed numerically. The effects of heat absorption, Joule heating and variable viscosity on momentum and heat transfer have been studied in detail. The conclusions of this study are as follows:

- The skin friction coefficient decreases that is the frictional force at the wall reduces over the whole boundary layer but the rate of heat transfer enhances in presence of heat absorption.
- The skin friction coefficient enhances and the rate of heat transfer reduces for higher values of the Joule heating parameter and temperature dependent viscosity parameter.
- In presence of heat absorption, the velocity of the fluid flow and the temperature distribution of the fluid decrease within the boundary layer. But the opposite results obtained for the effect of *J*.

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